Theory and Performance for Position and Gravity Survey with an Inertial System

James R. Huddle
Litton Industries, Woodland Hills, Calif.

As in conventional surveying, systematic errors and noise effects contaminate the survey results obtained with an inertial surveyor. Appropriate techniques of calibration can be employed to essentially eliminate the effects of systematic error. Sources of noise affecting survey performance arise from various phenomena within the inertial equipment, as well as from environmental influences such as the uncertainty in the Earth's gravity field. This paper discusses the theoretical basis for the control of error in the inertial surveyor position and gravity determinations as induced principally by noise or error parameter instability effects, which fundamentally limit performance. Theoretical estimates of inertial surveyor position and deflection of the vertical measurement performance for nominal inertial system noise source error parameters and gravity disturbance assumptions are presented.

Introduction

THE idea of position surveying with an inertial system has been under investigation for a considerable period of time. The factors that set the stage for the initial development of the inertial surveyor in the mid 1960's were the 1) fairly advanced state of development of inertial sensor and digital computer hardware technology; 2) discovery of optimal filtering and smoothing techniques for control of error in linear dynamic stochastic systems; 3) exploitation of systematic error compensation in real-time via software techniques; and 4) existence of an important need by the U.S. Army for a totally self-contained, portable black box that could provide accurate position and azimuth data rapidly for field operations.

The first theoretical results applying optimal estimation theory to the processing of error in system-computed velocity observed at vehicle stops as discussed in the following in performing position survey and azimuth laying were published in 1966.[1] Subsequent theoretical refinements and test results demonstrating this concept for "open traverse" survey have appeared over the past decade.[2-6] Improved results using optimal smoothing techniques[7] to process the errors in the real-time estimates of the position observed at "closure" of a survey traverse also have been demonstrated in the field.[8-11]

Theoretical investigations into the recovery of the change in the gravity vector have been performed in the past. Most of this work has dealt with deflection determination on a moving base at sea using high-density position and velocity measurements from aid sensors.[12-15] An alternate approach of using the error in system-computed velocity and the platform tilt relative to the local vertical observed at survey vehicle stops also has been investigated[16] and successfully field-tested.[17-19] This latter method achieves direct separation of gravity and vehicle acceleration via stopping the survey vehicle.

More recently, theoretical investigations and performance projections have been obtained for more exotic gravity-gradiometer-aided inertial system mechanizations.[20,21] These methods achieve continuous separation of gravity and vehicle acceleration during vehicle motion and offer potential for improved survey performance at some point in the future.

This paper provides a tutorial discussion of the theoretical principles that have been employed in the design of the inertial surveyor as it exists and is employed in the field today. The tutorial discussion of the position survey problem is new, as is the discussion of the principles employed in recovering the change in the deflection of the vertical along with the associated performance estimates.

To appreciate the key role that optimal estimation software has played in the development of the inertial surveyor, one must realize that the basic sensor that is being used was originally designed for aircraft navigation. Such equipment was only required to measure position change with an error that increases with time at a rate between 200 and 1000 m/h, as illustrated in Fig. 1. The adaptation of such a sensor to survey with a goal of submeter accuracy over several hours has required 1) development of employment strategies consistent with survey operations that are a gross departure from those encountered with fixed-wing aircraft, and 2) use of optimal estimation software to process the measurements that such operating strategies make available. The manner of using the inertial navigation system for position and gravity survey involves 1) periodically stopping the survey vehicle to mark or survey the point and to observe the error in computed velocity and the platform tilt relative to the local vertical, and 2) running a traverse between survey control points and stations where the gravity vector is known to observe the error in the computed position change and the estimated change in the vertical deflection.

In addition to the use of software optimal-estimation techniques to process the measurements just indicated, extreme care must be taken in the initial factory calibration of the inertial surveyor, with derived systematic error correction parameters being stored in the computer for real-time compensation. A presurvey calibration mechanization also is employed before system use to contend with those errors that are systematic on a single-traverse basis. The postsurvey smoothing programs can counteract some errors that are systematic on a single-traverse basis, but they also have utility in contending with any gradual deterioration of factory-calibrated systematic error parameters over a number of months of equipment operation.

Since the various principles employed in the control of systematic error in the inertial surveyor have been discussed previously,[4,7,8] this paper will concentrate on the effects of system error instability or noise sources on the position and

Received Oct. 14, 1977; revision received Dec. 21, 1977.
Index categories: Aerospace Technology Utilization; Analytical and Numerical Methods; Sensor Systems.
*Chief Scientist.

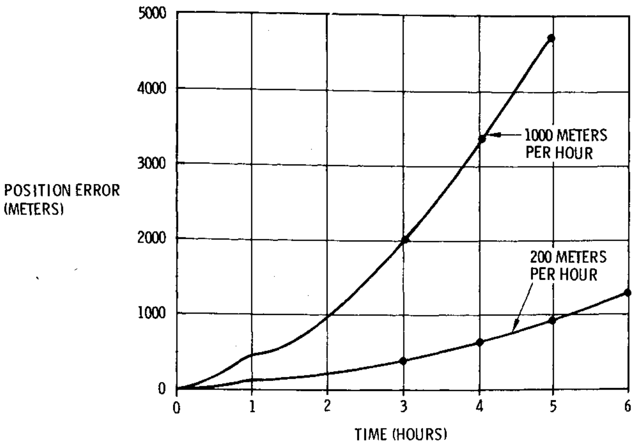
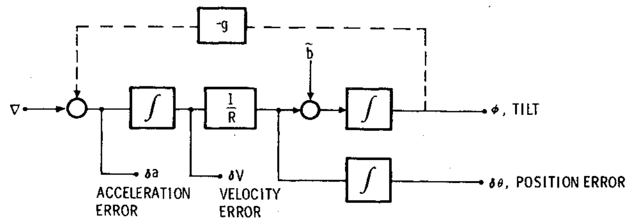


Fig. 1 Typical propagation rates for position error for an aircraft inertial navigation system.



- ACCELERATION ERROR NODE (AFTER LEVELING)

$\phi \approx \left[\frac{\nabla}{g}\right] \rightarrow \delta a \approx 0,$ ∇ DUE TO ACCELEROMETER BIAS/NOISE AND DEFLECTION OF THE VERTICAL

- TILT RATE NODE - RESIDUAL PLATFORM DRIFT RATE (UNCOMPENSATED)

$\tilde{b} \approx 0.001°/HR$

- RESIDUAL VELOCITY ERROR - DUE TO PLATFORM NONSTATIONARITY DURING ALIGNMENT

$\delta V_0 < 0.001$ fps

Fig. 2 Nominal system error state after an initial alignment.

gravity vector change measurements. Such error sources establish the accuracy limit to which the inertial surveyor can perform.

System Noise Sources

The sources of noise or error parameter instabilities associated with the inertial surveyor can be assigned to one of three major categories: 1) accelerometer measurement error; 2) platform drift rate; or 3) environmental effects. Measurement error instabilities associated with the accelerometer are induced by thermal or vibration effects during the course of a survey traverse. In addition, a noise with an exponential correlation characteristic is present in the accelerometer output even during quiescent operation. Accelerometer bias, constant residual scale-factor error, and sensitive-axis mechanical misalignment usually can be regarded as systematic errors whose effect essentially can be eliminated at traverse closure under certain traverse constraints.[7]

Platform drift rate instability also occurs because of vibration variation and thermal transients during a survey traverse. Additionally, a component of exponentially correlated noise is used to characterize gyro drift rate under quiescent conditions. The effect of constant platform drift rate due to precession axis misalignment, residual torquer scale factor error, and bias can be regarded as systematic error and eliminated at traverse closure if their effect is truly constant during the traverse.

The principal sources of noise associated with the environment in which the inertial surveyor operates are the unknown variations in the Earth's gravity field and the nonstationarity of the survey vehicle at stops where observations of the error in system-computed velocity are made. These latter observations are employed for controlling the growth in system errors during open traverse and are discussed in what follows.

Theory for Correction of Position Error with Velocity Error during Traverse

The idea of correcting the error in system-computed position using periodic observations of the error in system-computed velocity is fundamental in obtaining survey position accuracy using a medium-quality inertial sensor. To introduce this concept, the reader is referred to Fig. 2, which depicts schematically a simplified error block diagram of a Schuler-tuned inertial platform after an alignment and calibration has been performed at the initial point of the traverse. Three factors of importance should be noted relative to the initial error status of the system:

1) The output of the acceleration node δa is essentially null as the initial tilt of the platform relative to the reference ellipsoid ϕ and the deflection of the vertical have been brought into equilibrium by platform leveling.

2) The velocity error δV, which is the difference between true instantaneous velocity of the sensor and the computed velocity, is quite small depending on the level of nonstationarity of the vehicle at stops.

3) A small residual drift rate bias $\tilde{b}$ will exist at the tilt rate node as induced by accelerometer noise and drift rate instability present during calibration.

If one examines the effects of an initial velocity error of 0.001 ft/s, a constant platform drift rate of 0.001 deg/h, and a hypothesized exponential change from the equilibrium condition at the acceleration error node due to accelerometer bias instability and change in the vertical deflection due to vehicle position change of the form

$$\delta\epsilon = \{1 - \exp[-\alpha t]\}\nabla$$

where

α^{-1} = 0.5 h is the time constant of the exponential acceleration error change

∇ = 50 mgal is the maximum magnitude of the acceleration error change (1 mgal is 0.001 cm/s^2)

then the short-term solutions vs time for the velocity error δV and change in positon error $\Delta\delta P$, shown in Fig. 3, will be obtained. In this highly simplified example, it is seen that

$$\delta V \approx 0.5[\alpha\nabla + bg]t^2 + \delta V_0$$

$$\Delta\delta P \approx \delta V[t/3]$$

Consequently, if the survey vehicle is brought to a stop, making the velocity error observable, an approximate correction for the change in position error over the last travel interval would be

$$\Delta\widehat{\delta P} = \widehat{\delta V}[t/3]$$

where $\widehat{\delta V}$ is the observed velocity error at the stop. Furthermore, the velocity error and the output of the acceleration error node could be essentially nulled, so that the system reobtains its initial equilibrium condition except for a residual error in computed position due to the approximation of the aforementioned correction.

The preceding explanation ignores the fact that the acceleration measurement error and the platform drift rate change in an arbitrary manner during the vehicle travel period. If we represent the effects of such phenomena as zero initial condition, exponentially correlated acceleration measurement noise, then we can solve for the variances of the

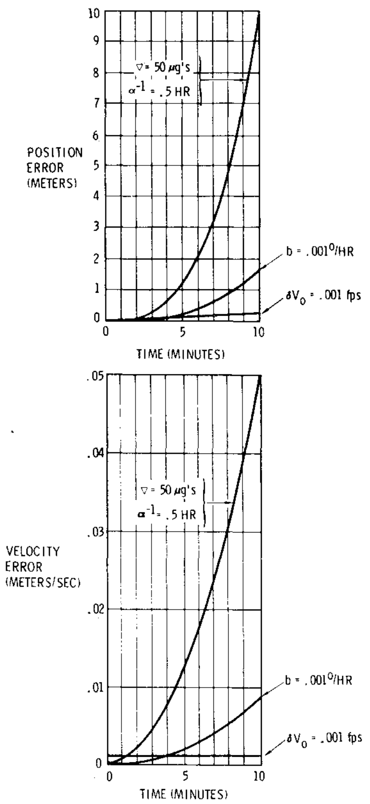
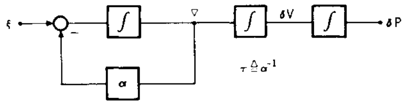


Fig. 3 Approximate short-term deterministic error propagation after initial alignment.

REPRESENT:

- ACCELERATION MEASUREMENT ERROR CHANGE
- INTEGRATED PLATFORM DRIFT RATE - OVER A TRAVEL PERIOD

AS ZERO INITIAL CONDITION EXPONENTIALLY-CORRELATED NOISE



- ACCELERATION MEASUREMENT ERROR VARIANCE

$$\left[\frac{\sigma_{\nabla}(t)}{\sigma_{\nabla}(\infty)}\right]^2 = \left[1 - \exp\left[-2\alpha t\right]\right]$$

- VELOCITY ERROR VARIANCE

$$\left[\frac{\sigma_{\delta V}(t)}{\tau \cdot \sigma_{\nabla}(\infty)}\right]^2 = \left[\left[1 + 2\alpha t\right] - \left[2 - \exp\left[-\alpha t\right]\right]^2\right]$$

- POSITION ERROR VARIANCE

$$\left[\frac{\sigma_{\delta P}(t)}{\tau^2 \sigma_{\nabla}(\infty)}\right]^2 = \left[\left[1 + \alpha t\right]^2 + \left[\alpha t\right]^2 \cdot \left[1 + \left(\frac{2}{3}\right)\alpha t\right] - \left[2\alpha t + \exp\left[-\alpha t\right]\right]^2\right]$$

- CORRELATION BETWEEN VELOCITY AND ACCELERATION ERROR

$$\rho_{\delta V \cdot \nabla}(t) = \frac{\left[1 - \exp\left[-\alpha t\right]\right]^2}{\left\{\left[1 - \exp\left[-2\alpha t\right]\right] \cdot \left[\left[1 + 2\alpha t - \left[2 - \exp\left[-\alpha t\right]\right]^2\right]\right\}^{1/2}}$$

- CORRELATION BETWEEN POSITION AND ACCELERATION ERROR

$$\rho_{\delta P \cdot \nabla}(t) = \frac{\left[1 - 2\alpha t \exp\left[-\alpha t\right] - \exp\left[-2\alpha t\right]\right]}{\left\{\left[1 - \exp\left[-2\alpha t\right]\right] \cdot \left[\left[1 + \alpha t\right]^2 + \left[\alpha t\right]^2 \cdot \left[1 + \left(\frac{2}{3}\right)\alpha t\right] - \left[2\alpha t + \exp\left[-\alpha t\right]\right]^2\right]\right\}^{1/2}}$$

- CORRELATION BETWEEN POSITION AND VELOCITY ERROR

$$\rho_{\delta P \cdot \delta V}(t) = \frac{\left[\alpha t - 1\right]^2 + \exp\left[-\alpha t\right]\left[\exp\left[-\alpha t\right] + 2\left(\alpha t - 1\right)\right]}{\left\{\left[\left[1 + 2\alpha t\right] - \left[2 - \exp\left[-\alpha t\right]\right]^2\right] \cdot \left[\left[1 + \alpha t\right]^2 + \left[\alpha t\right]^2 \cdot \left[1 + \left(\frac{2}{3}\right)\alpha t\right] - \left[2\alpha t + \exp\left[-\alpha t\right]\right]^2\right]\right\}^{1/2}}$$

Fig. 4 Approximate short-term stochastic error propagation.

errors in the acceleration measurement and computed velocity and position and also for the three statistical correlation coefficients between these errors. Such solutions are summarized in Fig. 4. The correlation coefficient of principal interest here is that between the change in position error and the velocity error which has a limit value of $\sqrt{15}/4$ for infinitesimally short travel periods and a limit value of a $\sqrt{3}/2$ for infinitely long travel periods.

From the theory of optimal estimation, if a perfect observation of a variable is available (e.g., δV) and the correlation coefficient between it and an unobserved variable (e.g., $\Delta\delta P$) is known, then the variance of the change in position error can be reduced using a perfect observation of the velocity error as

$$[\sigma^{+}_{\Delta\delta P}]^2 = r^2 [\sigma^{-}_{\Delta\delta P}]^2$$

where

$[\sigma^{+,-}_{\Delta\delta P}]$	= variance of the change in position error over the last travel period before and after correction, respectively
$r^2 \triangleq [1 - \rho^2]$	= position error change reduction factor
ρ	= correlation coefficient between the observed velocity error and the change in position error over the last travel period.

The factor r by which the (1σ) value of the change in position error can be reduced at a stop is the quadrant of a circle when plotted vs the magnitude of the correlation coefficient ρ, as shown in Fig. 5. At the limits of the correlation coefficient noted in Fig. 4, this implies an ability to remove, at survey stop points, 50% of the position error change for long travel periods and 75% of the position error change for short travel periods. It should be noted further that, as the travel period increases, the (1σ) value of position error change increases as $t^{5/2}$, so that with increasing travel periods the ability to counteract position error change decreases because of both the decrease in the correlation coefficient and the increase in the (1σ) error value itself.

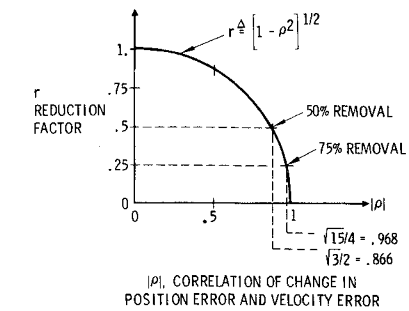


Fig. 5 Correction of change in position error with error in computed velocity observed at a stop.

It turns out that the residual error in the computed position change after correction at a stop tends to be uncorrelated from one stop to the next. Consquently, the (1σ) value of position error after correction at stops due to system noise effects tends to grow as the square root of the number of stops, or approximately as the integral of white noise.

Theory for Correction of Survey Position Error with Error in Computed Position at Traverse Closure

When the traverse is closed at the terminal point, the position error accumulated over the survey is observable and can be used to improve the preliminary estimates of position obtained at the intermediate survey points. It can be shown[7] that the effects of systematic accelerometer scale factor and constant pointing errors can be eliminated entirely if the surveyed positions of the traverse lie along an essentially straight line between the initial and terminal control points and the survey is performed at a constant rate with the same degree of system settling being obtained at each stop. This result implies, then, that the residual error in surveyed position at the stopping points will be due only to the system noise effects.

As noted previously, the error in surveyed position after velocity correction at stops due to system noise sources only propagates as the integral of white noise:

$$\delta P(t) = \int_0^t \xi(\mu)\, d\mu$$

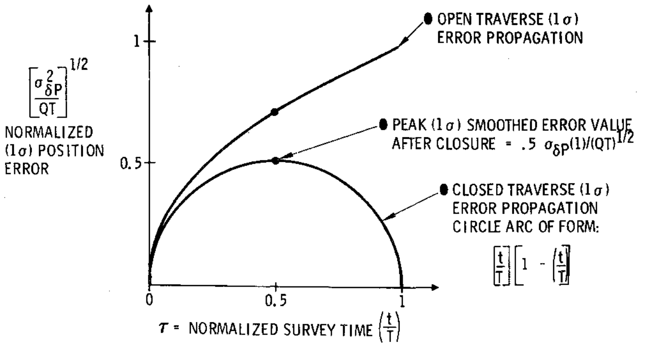


Fig. 6 Inertial survey position error propagation in open and closed traverse due to noise effects only.

where

$\delta P(t)$ = error in surveyed position after velocity correction at vehicle stops

$\xi(t)$ = white noise process

The optimal smoothed estimate of this survey position error at the intermediate points along the traverse using the position closure error $\delta P(T)$ is

$$\delta\hat{P}(t\,|\,T) = [t/T]\delta P(T)$$

which results in a residual error after smoothing with variance

$$\sigma^2_{\delta P(t|T)} = Qt\{1-[t/T]\}$$

where Q is the power spectral density of the approximating white noise process $\{\xi(t)\}$. The (1σ) value of the position error propagation in open traverse is depicted in normalized form in Fig. 6, together with the (1σ) value of this position error after traverse closure smoothing has been performed. This latter curve is seen to be a semicircle with a peak (1σ) value at the midpoint of the traverse, which is one-half the (1σ) value of open traverse position error at closure.

Survey Positioning Error Estimates

To obtain estimates of the positioning accuracy of the inertial surveyor, a covariance analysis based upon a noise model of the system was performed. The error parameter values employed in this model were 1) accelerometer noise of 1 mgal (1σ) with a 30-min correlation time; 2) platform drift rate of 0.001 deg/h (1σ) with a 2-h correlation time; 3) error in observing the error in computed velocity at a stop over a 10-s averaging period of 0.001 ft/s (1σ) with vehicle stopping periods of 1 min; and 4) deflection of the vertical of 10 arc-s (1σ) with a correlation distance of 20 n.m. The analysis was conducted for 1) three vehicle travel periods between stops of 3, 5, and 10 min; 2) two vehicle speeds of 55 and 184 km/h; and 3) survey times of up to 12 h duration.

The results of this analysis are presented in Figs. 7 and 8 for the 55- and 184-km/h speed cases, respectively. In each of the two figures, the three solid curves show how the (1σ) position error in feet for the open traverse increases as the travel period between stops increases. This occurs because of the decrease in the correlation between position error and observed velocity error at the stops and because of the increase in accumulated position error during the longer travel period. In comparing the results, it is seen that, at the higher speed, the position error is greater and increasingly so as the travel period increases. This occurs as more variation in the deflection of the vertical takes place in any one travel period, thereby lessening the certainty with which the observed first integral, velocity, can be related to the second integral, position.

The figures also show the peak (1σ) value of the position error after a 12-h closure in all cases and after 2-, 4-, and 8-h

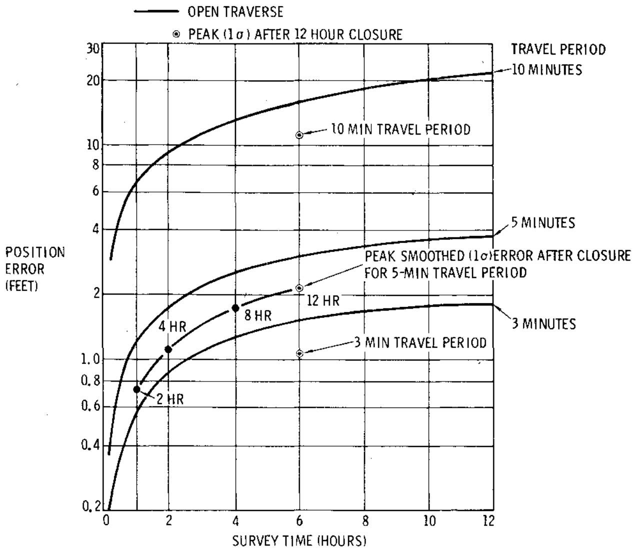


Fig. 7 Survey position error due to noise (55 km/h).

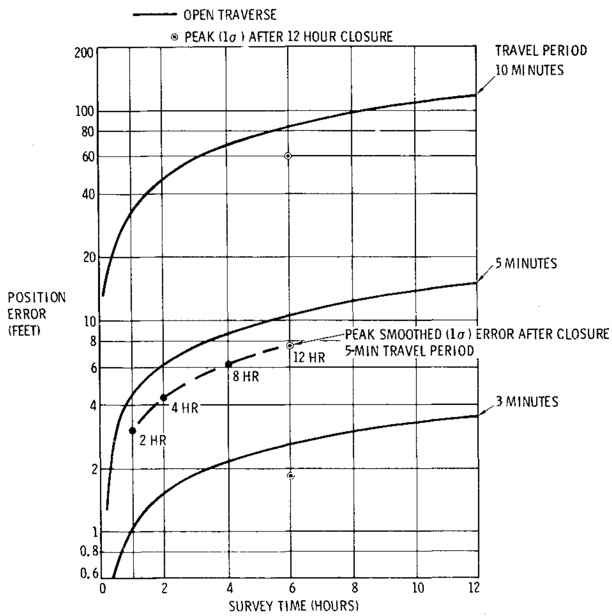


Fig. 8 Survey position error due to noise (184km/h).

closure for the case of a 5-min travel period. Analysis of these closure results indicates that they are in consonance with predictions based upon the approximate theory of the previous section.

Theory for Measurement of Change in Deflection of the Vertical

In theory, when a Schuler-tuned inertial platform is aligned, it is presumed that the level-axis accelerometers are tangent to the mathematical figure of the Earth (normal gravity field) which is assumed in the subsequent navigation computations. As leveling results in the level accelerometers being orthogonal to the local vertical, even in the absence of all other system error sources, this condition will not occur unless the initial deflection is known and employed appropriately during alignment. Once alignment is complete, again in theory, the gyros of a north-slaved platform are precessed so that the accelerometers remain in coincidence with the geodetic east, north, and vertical axes at the instantaneous platform position. The angular rate required to perform this function is derived from a knowledge of 1) local components of Earth rate about the platform axes which are

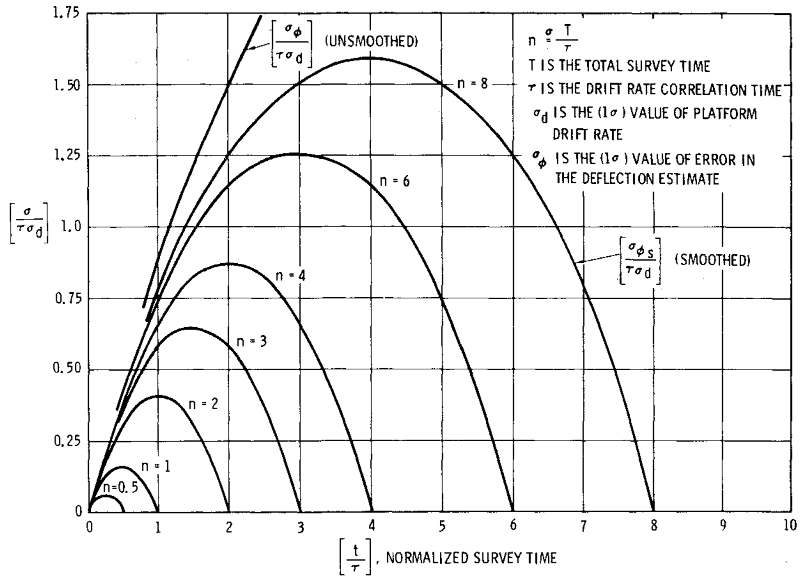


Fig. 9 Normalized error (1σ) in the estimated vertical deflection due to integrated, exponentially correlated drift rate noise vs normalized survey time with normalized total survey time as a parameter.

obtained using the system-computed latitude and azimuth of the platform; and 2) angular rate of change of position of the vehicle relative to the Earth which is obtained from the system-computed velocity divided by the local radii of curvature of the Earth in the plane of the level platform axes. In the absence of all other error sources, the change in the deflection of the vertical as the survey vehicle travels over the Earth's surface will induce platform tilt and error in system-computed velocity and position. At vehicle stops, however, the error in system-computed velocity is observable and can be used to remove the induced change in platform tilt, as well as the error in system-computed position. With this corrective action being taken, the level-axis accelerometers can be used to observe the change in the deflection of the vertical relative to the initial point of the traverse.

It turns out that, for state-of-the-art inertial equipment, the most serious source of error in the estimate of the change in the deflection of the vertical during a traverse is the platform drift rate. However, when a gravity surveying traverse is run between two stations where the deflections have been determined, then the observed error in the system-estimated deflection at closure can be employed to eliminate the effects of constant platform drift rate. Consequently, the principal contaminant in the estimate of the change in the deflection of the vertical after closure is the platform drift rate instability or noise that occurs during the traverse.

Deflection Survey Error Estimates

With a little thought, one concludes that, if the unstable platform drift rate could be characterized as white noise (relative to the total time to perform the survey), then, since the error in the estimated deflection change is its integral, the theory just developed for the character of the position error propagation in the open traverse as well as after traverse closure applies immediately because of the equivalence of the two problems. However, the case where the total time of the survey is not a large multiple of the correlation time of the gyro drift rate is of more general interest. For the reasonable assumption that the gyro drift rate noise is exponential in character, a solution to this more general problem, assuming smoothing after traverse closure, is equivalent to obtaining the smoothed estimate of the integral of correlated noise over the total survey time interval T. Here the smoothed estimate is obtained using only the observed value of the integral (e.g., the error in the deflection change estimate at closure) at the terminal time T.

The general solution to this problem in normalized form is presented in Figs. 9 and 10. One should observe that, as the

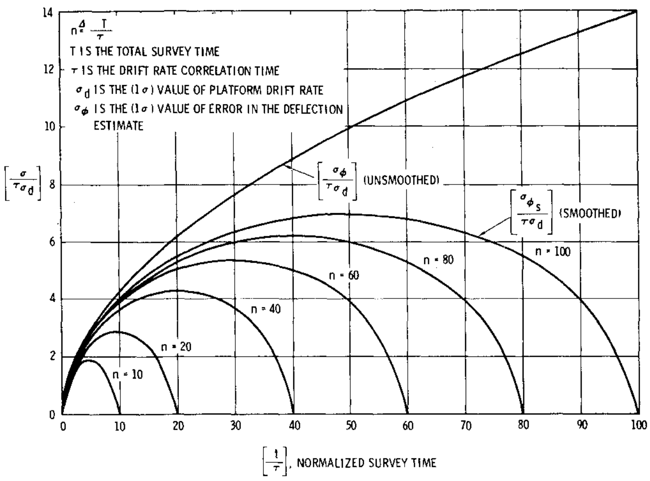


Fig. 10 Normalized error (1σ) in the estimated vertical deflection due to integrated, exponentially correlated drift rate vs normalized survey time with normalized total survey time as a parameter.

total survey time T becomes a large multiple of the drift rate correlation time τ (e.g., as n becomes large), the (1σ) value of the error in smoothed estimate of the deflection of the vertical approaches a semicircular arc, as in the case of position error discussed previously. As the drift rate correlation time becomes more significant relative to the total survey time, the (1σ) value of the error in the smoothed estimate of the deflection is parabolic in form when plotted vs the normalized survey time $[t/\tau]$.

Employing the general solutions of Figs. 9 and 10 for a specific case where $\sigma_d = 0.001$ deg/h is the drift rate (1σ) value, $\tau = 2$ h is the drift rate correlation time, and $T = 1$ h is the total survey time, then one finds, employing the curve $n = 0.5$ of Fig. 9, that the maximum (1σ) value of the error in determining the change in the deflection (due to drift rate noise only) is

$$\sigma_{\phi_s} = [\tau \sigma_d][0.0625] = 0.45 \text{ arc-s}$$

which occurs at the midpoint ($t = 30$ m) of the survey. The (1σ) value of the estimate in the deflection at closure before smoothing is much larger, being

$$\sigma_{\phi} = [\tau \sigma_d][0.375] = 2.7 \text{ arc-s}$$

Theory for the Measurement of Change in the Gravity Disturbance

At a vehicle stop, the vertical accelerometer output is the sum of the gravity intensity, accelerometer measurement error, and accelerative effects due to vehicle nonstationarity. Correction of the average value of such measurements with the computed theoretical value of gravity at the stop and an estimate of the inseparable sum of accelerometer measurement error and the gravity disturbance at the initial point yields an estimate of the change in the gravity disturbance relative to the initial point.

Measurement averaging reduces the effects of short-term accelerometer noise and vehicle motion. Since the error in surveyed elevation can be kept small, the error in the computed theoretical gravity usually is minor (0.3 mgal/m). The principal error in surveying the change in the gravity disturbance is due to longer-term accelerometer measurement error instabilities, which for the current equipment approximate 1 mgal.[17,19]

Summary

This paper has presented in tutorial fashion the theoretical principles upon which the design of the inertial position and gravity surveying system is based. Estimates as to the limits of system performance due to noise, assuming nominal existing inertial equipment and a model of the uncertainty in the gravity field, also have been provided.

The fact that such accuracy is obtainable with inertial equipment that is readily available today has been met with some astonishment even in the inertial community. This phase, however, is passing, as the confirming test results from numerous users are being documented. The accuracy of results should improve naturally in the future as more accurate inertial components are used, improved methods of system use are perfected, and more comprehensive approaches to off-line data processing are developed. However, the main thrust of importance probably should be in the area of optimally organizing the survey work so that the great speed with which the inertial surveyor can perform its function is capitalized upon fully. Such concepts for employing the survey machine are evolving and soon will appear in the open literature on inertial survey.

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